MATH-818 Theory of Ordinary Differential Equations

Credit Hours: 3-0

Prerequisite: None

Course Objectives: Modern technology requires a deeper knowledge of the behavior of real physical phenomena. Mathematical models of real-world phenomenon are formulated as algebraic, differential or integral equations (or a combination of them). After the construction of equations, the study of their properties is necessary. At this stage the theory of ordinary differential equations plays a significant role. In this course we shall discuss the stability theory and phase-plane analysis of dynamical systems, bifurcation theory, Non-oscillation and oscillation theory and the existence theory of differential equations.

Detailed Course Contents: General theory of linear equations, Homogeneous Linear Equations with periodic coefficients: Floquet multipliers, Floquet Theorem, Stability of linear equations, Stability of linear equations by Lozinskii measures, Perturbations of linear equations, Lyapunov function method for autonomous equations, Lyapunov function methodfor nonautonomous equations, General theory of autonomous equations, Poincar´e– Bendixson Theorem, Periodic solutions and orbital stability, Basic concepts of bifurcation theory, One-dimensional bifurcations for scalar equations, One- dimensional bifurcations for planar systems, Hopf bifurcations for planar systems, Second-order linear equations, Self- adjoint second-order differential equation, Cauchy Function and Variation of Constants Formula, Sturm-Liouville problems, Zeros of solutions and disconjugacy, Factorizations and recessive and dominant solutions Oscillation and non-oscillation, Applications of the CMT to BVPs, Lower and upper solutions, Nagumo condition, Lipschitz condition and Picard-Lindelof Theorem, Equicontinuity and the Ascoli-Arzela Theorem, Cauchy-Peano Theorem, Extendability of solutions, Basic Convergence Theorem, Continuity of solutions with Respect to ICs, Kneser's Theorem. Differentiating solutions with respect to ICs, Maximum and minimum solutions.

Course Outcomes: Students are expected to understand topics such as stability theory, bifurcation theory, phase-plane analysis of dynamical systems, and existence theory of differential equations.

Text Books:

Theory of Differential Equations, W. G. Kelley, A. C. Peterson Springer, 2010.

Qingkai Kong, A Short Course in Ordinary Differential Equations, Springer 2014 (Referred as QK)

Reference Books:

1. Ordinary differential equations I.G.Petrovski, Dover Publications, Inc., 1973
2. Theory of ordinary differential equations, Coddington E.A. and Levinson, New York:McGraw-Hill, 1955.
3. Nonlinear Ordinary Differential Equations, D. W. Jordan and P. Smith, Oxford University Press, 2007

ASSESSMENT SYSTEM

Nature of assessment	Frequency	Weightage (%age)
Quizzes	Minimum 3	10-15
Assignments	-	5-10
Midterm	1	25-35
End Semester Examination	1	40-50
Project(s)	-	10-20

Weekly Breakdown		
Week	***Section***	***Topics***
1	Ch. 1	First order equations: Existence, Bifurcation, Stability

2	2.3	The Matrix Exponential Function, Putzer Algorithm, Lozinski measure
3	2.5	Homogeneous Linear Equations with periodic coefficients: Floquet multipliers, Floquet Theorem
4	3.1-3.3	Phase plane diagram, homoclinic orbits, Hamiltonian systems
5	3.4, 3.5	Stability of nonlinear systems, Semi-group property, Lyapunov function method forautonomous and non-autonomous equations, Linearization of nonlinear systems
6	3.6	Existence and nonexistence of periodic, Solutions, Poincar´e–Bendixson Theorem,Bendixson-Dulac Theorem, Li´enard's Theorem
7	3.7	Three-dimensional systems
8	5.1, 5.2 (QK)	Basic concepts of bifurcation theory, One-dimensional bifurcations for scalarequations
9	**Mid Semester Exam**	
10	5.3, 5.4 (QK)	One-dimensional bifurcations for planar systems, Hopf bifurcations forplanar systems
11	5.1,5.2	Self-adjoint second-order differential equation: Basic concepts
12	5.3	Cauchy Function and variation of constants formula
13	5.4	Sturm-Liouville problems
14	5.5	Zeros of solutions and disconjugacy
15	5.6 ,5.7	Factorizations and recessive and dominant solutions, The Riccati Equation,
16	5.9	Green Function, Contraction Mapping Theorem (handouts)
17		Review
18	**End Semester Exam**	